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A Review of Copula Methods for Measuring Uncertainty in Finance and Economics

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ABSTRACT

This paper reviews copula methods used for economic and finance. Copula allows researchers to relax the traditional linear model assumptions so that researchers can specify the marginal distributions and look at the dependence structure linking the marginal distributions to form a joint distribution. In this review, we focus on the copula dynamic correlation coefficient, copula directional dependence and copula applications in economics and finance.

Key words : Copula, Directional dependence, Dynamic correlation coefficient, Econometrics

1. Introduction

During this COVID-19 pandemic period, the level of uncertainty has been increasing rapidly so that people have purchased more commodity, security or foreign currency such as gold, silver, stock and even cryptocurrency in their financial portfolio for risk management. Financial statistics and econometrics research is drawing more attention nowadays. Kim et al. [1] studied the relationships of financial markets such as cryptocurrency, stock and gold by using copulas. Because of relaxing the assumptions of normality, linearity and independence, copulas have been popular in the research areas of econometrics and finance over the last two decades [2]. Haseb [3] mentioned that a copula method is useful to fit the model by the maximum likelihood method so that the estimator attains efficiency. Especially, a copula-based approach to analyze financial time series has been popular because of the following attractive properties. First, due to Sklar's theorem [4], the appropriate marginals for the components of a multivariate joint distribution can be selected freely and then linked through a suitable copula so that the dependence structure may be modeled independently of the marginal distributions. Second, copulas are invariant under increasing and continuous transformations. Third, copulas do not require the marginals to be elliptically distributed, unlike correlations. Masarotto and Varin [5] proposed the Gaussian copula marginal regression (GCMR), and Guolo and Varin [6] proposed a beta regression model to analyze bounded time series. Guolo and Varin's method allowed the direct interpretation of the regression parameters on the original response scale, while properly accounting for the heteroscedasticity typical of bounded variables. The serial dependence is modeled by a Gaussian copula, with a correlation matrix corresponding to a stationary autoregressive and moving average process. With the GCMR method, Kim and Hwang [7] and Kim and Hwang [8] proposed the copula directional dependence by asymmetric generalized autoregressive conditional heteroscedasticity (GARCH) and stochastic volatility models. For the applications of the copula directional dependence model to cryptocurrency, Hyun et al. [9] proposed the copula directional dependence by neural networks model and applied

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it to five major cryptocurrencies, daily data consisting of Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC), Stella (XLM), and Ripple (XRP). With the GCMR method, Kim et al. [10] investigated a causal interpretation with board characteristics in corporate finance, and Kim et al. [11] modeled corporate bond yield spreads.

Engle [12] and Bollerslev [13] developed the GARCH model which has been the most popular volatility model over the last three decades, and the volatility is considered as deterministic conditionally on past information. Engle [14] proposed multivariate GARCH models for dynamic conditional correlation (DCC). Patton [15] also extended the GARCH model to a copula-based GARCH model for financial time series. Using an electronic trading platform to trade financial assets including common stocks, options, mutual funds, and fixed income investments, many consumers have easily traded their common stocks every second or minute through an electronic trading brokerage account. Engle [16] defined ultra-high-frequency as a full record of transactions and their associated characteristics, and Alexander [17] described high-frequency data as real-time tick data. High-frequency time series are now prevalent in financial data. This growth has been driven by the increasing availability of such big data, the technological advancements that make high-frequency trading strategies possible, and the need for practitioners to analyze this data [18]. The development of statistical and econometric methods for analyzing high-frequency financial data has grown exponentially. Hörmann et al. [19] and Aue et al. [20] developed the functional autoregressive conditional heteroscedasticity (fARCH) and functional GARCH (1,1) models that are empirically relevant for analyzing intraday volatilities of the high-frequency time series of tick-by-tick price changes. Yoon et al. [21] and Yoon et al. [22] applied the Hörmann et al. [19] fARCH model and the Aue et al. [20] fGARCH model to the Korea composite stock price index (KOSPI) and Hyundai motor stock high-frequency time series. Kim and Jung [24] also studied the relationship between oil price and exchange rate using functional data analysis and copulas. Yoon et al. [23] reviewed functional principal component analysis for volatility from high-frequency time series via the R-function. Kim and Hwang [25] also considered copula directional dependence with ultra high-frequency stock prices by employing the fARCH model, which is a functional statistical model for time series data proposed by [19], so that Kim and Hwang [25] could propose a copula fARCH directional dependence for intraday volatility with high-frequency financial data. Alqawbaa et al. [26] also considered the copula directional dependence of discrete time series marginals. Kim et al. [27] applied the copula directional dependence of discrete time series to the patent keyword analysis of the Apple technology company. The remainder of this review paper is organized as follows: Section 2 describes copula methods, copula dynamic conditional correlation (DCC) GARCH and copula directional dependence. In Section 3, we show the real data illustration with five major US stocks. Section 4 shows the list of copula applications in economics and finance. Finally, conclusions are presented in Section 5.

2. Copula Method

This section introduces definitions of copulas, the copula dynamic (time-varying) correlation coefficient (DCC) GARCH, and copula directional dependence.

2.1 Copula

A copula is a multivariate distribution function defined on the unit $[0,1]^n$, with uniformly distributed marginals. In this paper, we focus on a bivariate (two-dimensional) copula, where $n = 2$. Sklar [4] shows that any bivariate distribution function, $F_{XY}(x, y)$, can be represented as a function of its marginal distribution of X and Y, $F_X(x)$ and $F_Y(y)$, by using a two-dimensional copula $C(\cdot,\cdot)$. More specifically, the copula may be written as

$$F_{XY}(x, y) = C(F_X(x), F_Y(y)) = C(U, V),$$

where U and V are the continuous empirical marginal distribution functions $F_X(x)$ and $F_Y(y)$, respectively. Note that U and V have uniform distribution $U(0,1)$.

Definition 2. A 2-dimensional copula is a function C: $[0,1]^2 \rightarrow [0,1]$ with the following properties:

1. For all $(U_1, U_2) \in [0,1]^2$, $C(U_1, U_2) = 0$ if at least one coordinate of (U_1, U_2) is 0;
2. $C(U_1, 1) = U_1$ and $C(1, U_2) = U_2$ for all $U_i \in [0,1]$, $(i = 1,2)$;
3. C is 2-increasing, (see [28]).

2.2 The copula DCC-GARCH

The time-varying correlation of financial data has been more important after the COVID-19 pandemic than before the COVID-19 pandemic. It can be analyzed by the DCC-GARCH model. Tse and Tsui [29] proposed a multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations.

Forecasting for US stock market volatility, Kim et al. [30] proposed a linear time varying regression with a DCC-GARCH model for volatility. To improve the linear time varying regression with DCC-GARCH model, Kim and Jung [31] proposed a linear time varying regression with copula-DCC-GARCH model for volatility. Kim and Jung [32] proposed a directional time-varying partial correlation with the Gaussian copula-DCC-GARCH model. For neuroscience research, Lee and Kim [33] proposed a dynamic functional connectivity analysis of resting-state functional magnetic resonance imaging (fMRI) based on copula time-varying correlation and showed that the copula-DCC-GARCH model is more efficient than the DCC-GARCH model to detect neuronal activation with fMRI data. Lee and Kim [34] also proposed a dynamic functional connectivity analysis based on time-varying partial correlation with a copula-DCC-GARCH model. Kim et al. [1] showed that the copula-DCC-GARCH model has better efficiency and computation advantage than the DCC-GARCH model with high volatility financial data.

In this paper, we briefly review the DCC model of Engle [14] where the correlation matrix is time varying, and the covariance matrix can be decomposed into:

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R} \mathbf{D}_t = \rho_{ij}\sqrt{h_{iit}h_{jjt}}, \tag{1}$$

where $\mathbf{D}_t = diag(\sqrt{h_{11,t}}, \ldots, \sqrt{h_{nn,t}})$ containing the time-varying standard deviations is obtained from GARCH models. The DCC model in [14] has the following structure:

$$\mathbf{R}_t = diag(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t diag(\mathbf{Q}_t)^{-1/2}, \tag{2}$$

where

$$\begin{aligned}\mathbf{Q}_t &= \bar{\mathbf{Q}} + a(z_{t-1}z'_{t-1} - \bar{\mathbf{Q}}) + b(\mathbf{Q}_{t-1} - \bar{\mathbf{Q}}) \\ &= (1-a-b)\bar{\mathbf{Q}} + az_{t-1}z'_{t-1} + b\mathbf{Q}_{t-1},\end{aligned} \tag{3}$$

where $a, b \geq 0$ such that $a + b < 1$ to ensure stationarity and positive definiteness of $\mathbf{Q}$. $\bar{\mathbf{Q}}$ is the unconditional variance-covariance matrix of the standardized errors z_t.

Bodnar and Hautsch [35] proposed a copula-DCC-GARCH. The time-varying conditional correlation in the copula framework with the elliptical copulas is an extension of the DCC model. Let $\mathbf{r}_t = (r_{it}, \cdots, r_{nt})$ be a $n \times 1$ vector of asset returns, and it follows a copula GARCH model with joint distribution given by:

$$F(\mathbf{r}_t|\boldsymbol{\mu}_t, \mathbf{h}_t) = C(F_1(r_{1t}|\mu_{1t}, h_{1t}), \cdots, F_n(r_{nt}|\mu_{nt}, h_{nt})) \tag{4}$$

where F_i and C are the conditional distribution and the copula function, respectively.

The conditional mean $E[r_{it}|\Im_{t-1}] = \mu_{it}$ is a linear function of its one-lag past returns, and it follows an autoregressive moving average (ARMA) model of order 1 process. The conditional variance h_{it} follows a GARCH (1,1) process based on model selection criteria, such as the Akaike information criterion (AIC) and the Bayesian information criterion (BIC). They are defined as:

$$r_{it} = \mu_{it} + \theta_1(r_{it-1} - \mu_{it}) + \theta_2\epsilon_{it-1}^2 + \epsilon_{it},\ \epsilon_{it} = \sqrt{h_{it}}z_{it} \tag{5}$$

$$h_{it} = \omega + \alpha_1\varepsilon_{it-1}^2 + \beta h_{it-1}, \tag{6}$$

where z_{it} are i.i.d. random variables, which conditionally follow Johnson's reparametrized SU distribution, $z_{it} \sim JSU(\mu, \sigma, \nu, \tau)$ (see [36] in detail). In the four parameters, μ, σ are the mean and standard deviation for all values of the skew and shape parameters ν and τ, respectively. The dependence structure is modeled using elliptical copulas with conditional correlation $\mathbf{R}_t$ and constant shape parameter τ. The conditional density with a Gaussian copula and a Student-*t* copula, respectively, are given by:

$$c_t(U_{it}, \cdots, U_{nt}|\mathbf{R}_t) = \frac{f_t\left(F_i^{-1}(U_{it}), \cdots, F_i^{-1}(U_{nt})|\mathbf{R}_t\right)}{\prod_{i=1}^{n} f_i\left(F_i^{-1}(U_{it})\right)} \tag{7}$$

$$c_t(U_{it}, \cdots, U_{nt}|\mathbf{R}_t, \tau) = \frac{f_t\left(F_i^{-1}(U_{it}), \cdots, F_i^{-1}(U_{nt})|\mathbf{R}_t\right)}{\prod_{i=1}^{n} f_i\left(F_i^{-1}(U_{it})\right)}, \tag{8}$$

where $U_{it} = F_{it}(r_{it}|\mu_{it}, h_{it}, \nu_t, \tau_i)$ is the probability integral transformed values by F_{it} estimated via the GARCH process, and $F_i^{-1}(U_{it}|\tau)$ represents the quantile transformation. Finally, the joint density of the estimation is defined by:

$$\begin{aligned}&f(\mathbf{r}_t|\boldsymbol{\mu}_t, \mathbf{h}_t, \mathbf{R}_t, \tau) \\ &\quad = c_t(u_{it}, \cdots, u_{nt}|\mathbf{R}_t, \tau) \prod_{i=1}^{n} \frac{1}{\sqrt{h_{it}}} f_{it}(z_{it}|\nu_i, \tau_i).\end{aligned} \tag{9}$$

We can estimate each conditional correlation via the function **cgarchspec** command in the R package *rmgarch* implementing the Gaussian and Student-*t* copulas.

For the application data analysis in Section 3, we applied the copula-DCC-GARCH model to five major S&P 500 stock datasets. In particular, we employ a Gaussian copula in order to estimate the conditional covariance matrix. The copula-DCC-GARCH based on the Gaussian copula is called hereafter "GCTV".

2.3 Copula directional dependence

Sungur [37] proposed copula directional dependence with an asymmetric variant of the farlie-gumbel-morgenstern copula model developed by [38]. Kim et al. [39] applied the Sungur [37] copula directional dependence to histone gene-gene interaction, but there is a lack of fit to detect the directional dependence. To rectify this limitation, Kim and Kim [40] proposed directional dependence with the asymmetric multivariate copula functions proposed by [41] and [42], which are new families of copulas whose members are asymmetric by multiplying two symmetric copulas. Kim and Kim [40] had the quasi monte carlo method computation difficulty and model selection issue about which copula functions were the better fit to data among so many combinations of symmetric copulas. To simplify the copula directional dependence method for a practical purpose and easy-to-use extension to multivariate regression settings, Kim and Hwang [7] proposed a new directional dependence by using the Gaussian copula beta regression model. By using the Gaussian copula marginal regression (GCMR) of [5], Kim and Hwang [7] estimated the parameter of β_x for the $\text{logit}(\mu_t) = x_t^T \beta_x$. The inference is performed through a likelihood approach. Computation of the exact likelihood is possible only for continuous responses. Otherwise the likelihood function is approximated by importance sampling. Details of likelihood computations are discussed in [6]. Kim and Hwang [7] used a beta logit function with one continuous covariate by utilizing the Gaussian copula regression model. Before applying a Gaussian copula beta regression model with a single continuous covariate to financial data, Kim and Hwang [7] preprocessed the financial data exhibiting conditional heteroscedasticity to white noise process by employing an asymmetric GARCH (p,q) model to generate the GARCH standardized residuals, ϵ_{1t} and ϵ_{2t}. Through doing this procedure, Kim and Hwang [7] tried to avoid the serial dependence in the component time series [43]. After making the ϵ_{1t} and ϵ_{2t} from each asymmetric GARCH model, and transforming the two sets of residuals into two uniform distributions, U_t and V_t, in [0,1], Kim and Hwang [7] performed the directional dependence by the Gaussian copula marginal beta regression model.

Kim and Hwang [8] also proposed a copula directional dependence with the stochastic volatility (SV) model. There are two classes of models that are often used to estimate and forecast unobserved volatility in asset returns. The first model is the GARCH model by [12] and [13], and the second model is the SV model by [44]. The difference between the two most well-known models is that the SV model assumes two error processes, while the GARCH model allows for only a single error term. It means that the SV model can provide a better in-sample fit to financial data, see [45]. On the other hand, the SV model parameters are not always easy to estimate, while GARCH parameters can easily be estimated using maximum likelihood. To overcome the computation difficulty of the SV model, Kastner [46] developed the "stochvol" R package for dealing with stochastic volatility in time series. The "stochvol" R package utilizes Markov Chain Monte Carlo (MCMC) samplers to conduct inference by obtaining draws from the posterior distribution of parameters and latent variables, which can then be used for predicting future volatilities.

The purpose of the Kim and Hwang [8] copula directional dependence with the SV model was twofold as follows. The first goal was to propose an alternative method which is more efficient than the Kim and Hwang [7] copula directional dependence with an asymmetric GARCH in terms of the percent relative efficiency (RE (%))) of the absolute bias and the mean squared error. The second goal was that, with a bivariate copula distribution and the result of the copula directional dependence with SV, Kim and Hwang [8] showed the better forecasting performance of SV with the transformed conditional data, which is the effect variable given by the cause variable. In order to avoid the serial dependence in the component time series, Kim and Hwang [8] preprocessed the financial data exhibiting conditional heteroscedasticity to white noise process by employing the Bayesian SV model to make two mean standardized residuals, ϵ_{1t} and ϵ_{2t}, from each SV model fit by using the R package "stochvol" resid command. These two mean standardized residuals were then transformed to two uniform distributions, U_t and V_t, in [0,1] to apply the copula direction dependence to data. Like [7], Kim and Hwang [8] also assumed that U_t given $V_t = v_t$ follows a beta distribution $Beta(\mu_t, \kappa_t)$ parametrized in terms of the mean parameter $0 < \mu_t < 1$ and the precision parameter $\kappa_t > 0$ and denoted by $F(U_t; \theta)$ the cumulative distribution function of a beta random variable of mean $\mu_{U_t} = E(U_t|v_t)$. Dependence of the response U_t on the covariate v_t is obtained by assuming a logit model for the mean parameter, $\text{logit}(\mu_{U_t}) = x_t^T \beta_x$, where β_x is a 2-dimensional vector of coefficients.

$$\text{logit}(\mu_{U_t}) = \log[\frac{\mu_{U_t}}{1-\mu_{U_t}}] = \beta_0 + \beta_1 v_t, \text{ where } t = 1, \dots, n, \tag{10}$$

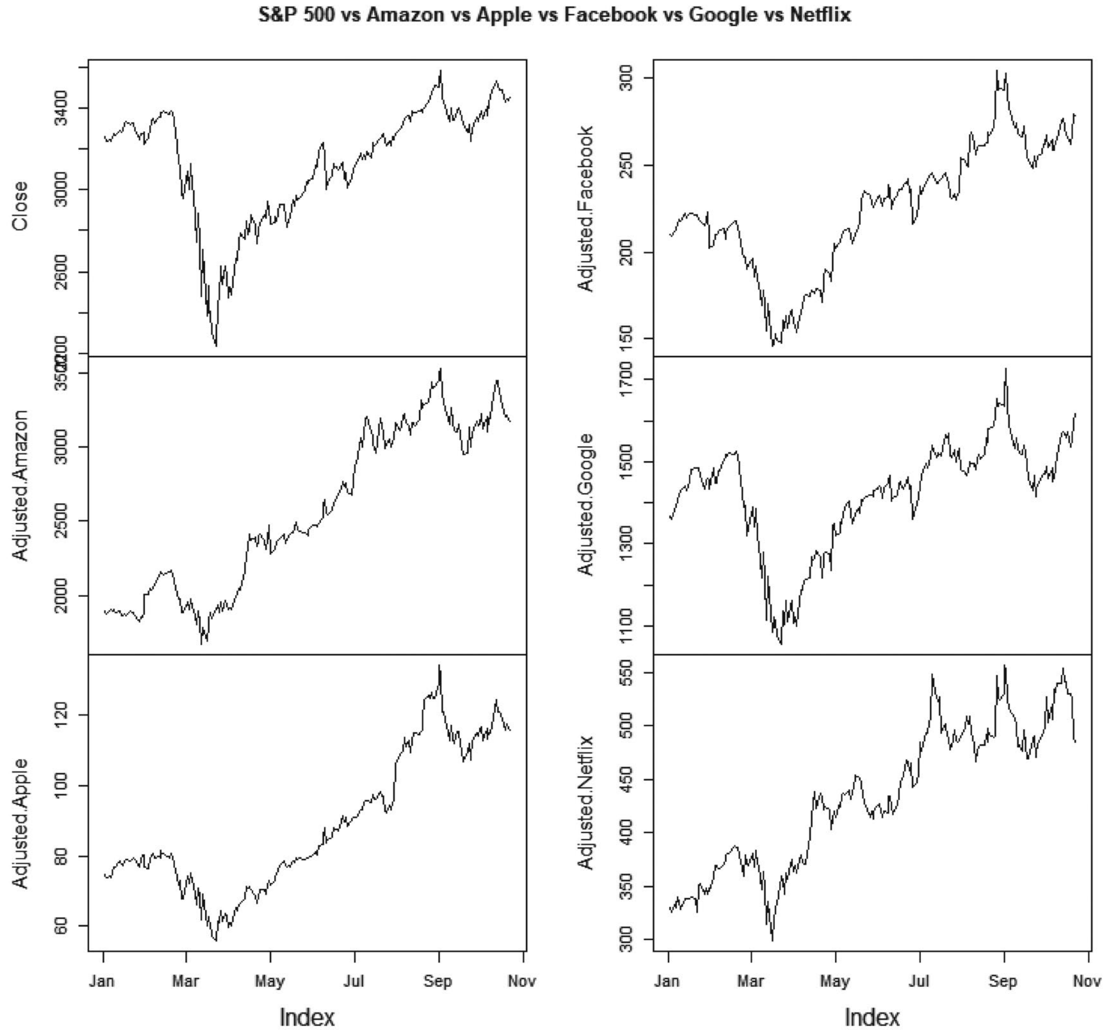

Fig. 1. S&P 500 and FAANG stock prices (period: January 3rd, 2020 to October, 22nd, 2020).

where $\mu_{U_t} = E(U_t|v_t) = \frac{\exp(\beta_0+\beta_1 v_t)}{1+\exp(\beta_0+\beta_1 v_t)}$ and $\kappa_{U_t} = 1 + \exp(\beta_0 + \beta_1 v_t)$ with the correlation matrix of the errors corresponding to the white noise process.

$$\rho^2_{V_t \to U_t} = \frac{Var(E(U_t|v_t))}{Var(U_t)} = 12Var(\mu_{U_t}) = 12\sigma^2_U, \quad (11)$$

where V_t is a uniform random variable in $[0,1]$, and we denote $\sigma^2_U = Var(\mu_{U_t})$.

To compute the estimate of $\rho^2_{V_t \to U_t}$, we used the "GCMR" R package [47] and choose a beta marginal distribution to find the estimates of (β_0, β_1) from Gaussian marginal regression. With these estimates of (β_0, β_1) and the covariate v_t, we compute $E(U_t|v_t) = \frac{\exp(\beta_0+\beta_1 v_t)}{1+\exp(\beta_0+\beta_1 v_t)}$ and then calculate $Var(E(U_t|v_t))$ and $Var(U_t)$. With these computed values, we compute the estimate of $\rho^2_{V_t \to U_t}$ in the equation (11).

The robustness of the copula directional dependence with SV model [8] was checked by two-sided randomization test and bootstrap 95% confidence interval.

3. US Stock Real Data Analysis

The COVID-19 pandemic has affected global economics severely, but global stock markets including the Korean stock market have been bullish since March 19th, 2020. Especially, the technology companies' stocks in the US stock market hit a record high price at the end of August, 2020. We chose FAANG stocks, which are well-known, and the most valuable American technology companies on the S&P 500 : Facebook

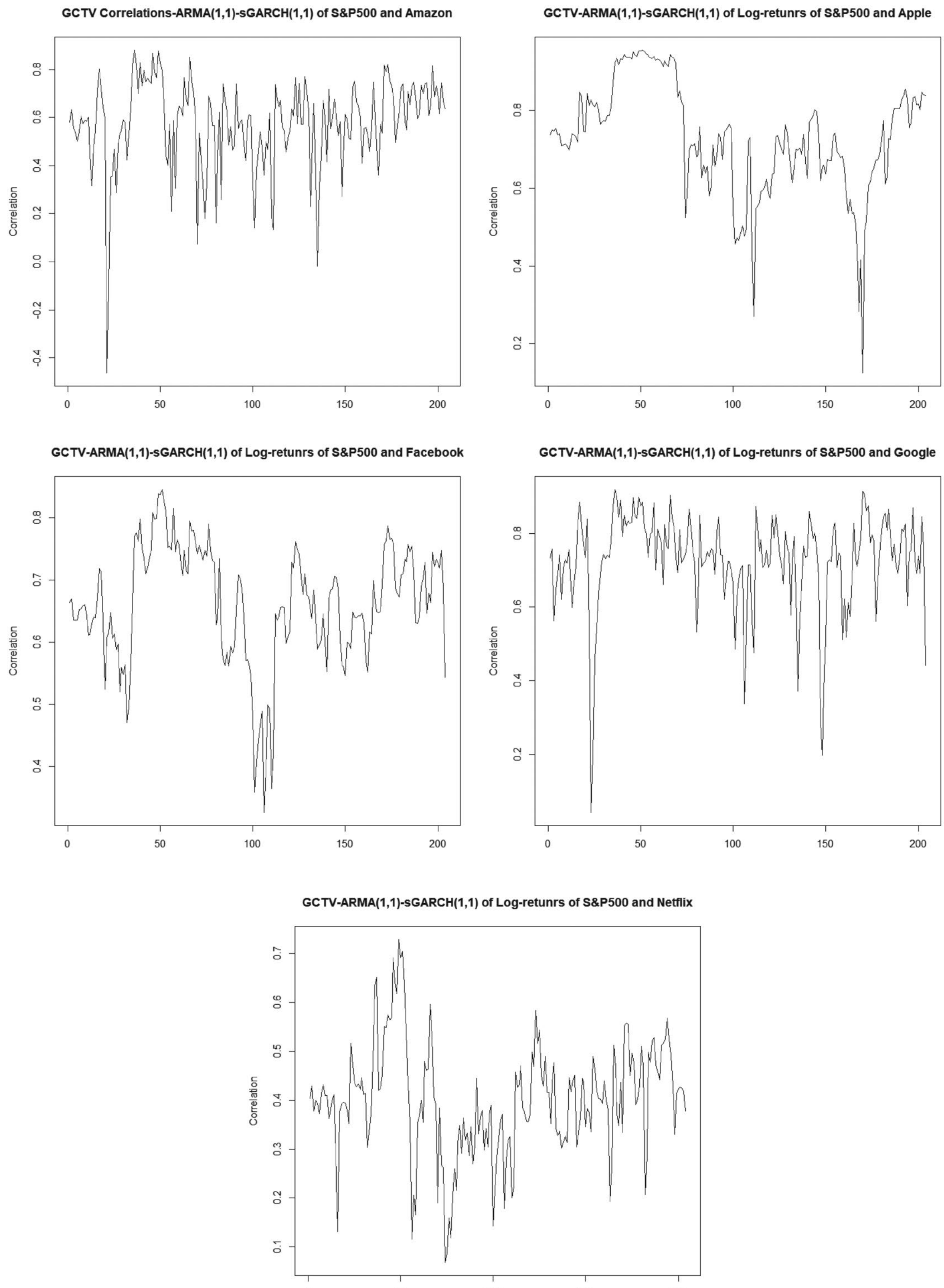

Fig. 2. Gaussian copula dynamic (time varying) correlation coefficients (period: January 3rd, 2020 to October, 22nd, 2020).

(FB), Amazon (AMZN), Apple (AAPL), Netflix (NFLX) and Alphabet (GOOG) (formerly known as Google). Consumers have heavily purchased the five FAANG stocks because FAANG stocks are widely known and are good return investment stocks in US stocks, with a combined market capitalization of over \$4.1 trillion as of January 2020. A Yahoo finance

(October 25th, 2020) report commented that the Big Tech stocks run-up has cooled somewhat in the months since these companies last reported earnings results over the summer in the year of 2020. Since July 30, Amazon (AMZN) underperformed the broader market, rising 5% versus the S&P 500's 6.8% gain. Alphabet (GOOG, GOOGL) tracked about in-line with the S&P 500, while Facebook (FB) and Apple (AAPL) each gained about 19% over that period.

We want to look at the relationship among FAANG stocks during this pandemic period (January 2nd, 2020 to October 22th, 2020). In Fig. 1, we can see the stock market crash on March 18th, 2020 because of COVID-19. Since then, not only the US stock market but also world stock markets including the Korean stock market have been bullish until now. We are skeptical of a US stock market bubble and worry about another stock market crash in the near future. Fig. 2 shows the Gaussian copula time varying correlations with log returns of S&P 500 with Amazon, Apple, Facebook, Google and Netflix. We can notice that the Gaussian copula time varying correlations with log returns of S&P 500 with Amazon and Apple have a high correlation in October, 2020. But recently, the Gaussian copula time varying correlations with log returns of S&P 500 with Facebook, Alphabet (GOOG, Google) and Netflix have been lower in October, 2020. These FAANG stocks have two different pattern groups against S&P 500 because the US Justice Department launched a landmark antitrust civil action against Alphabet (GOOG) and Facebook recently, and these two companies, CEOs testified before the US Senate commerce committee on October 28th, 2020. We also performed the [8] copula directional dependence with SV model with these log returns of S&P 500 with four big tech companies (Amazon, Apple, Facebook, Google). Fig. 3 shows that the four big tech companies give a higher directional dependence to S&P 500 in terms of the log returns of market and stock prices. We have two interesting findings from the US stock market during this COVID-19 pandemic period. First, the log-returns of Amazon stock affected the other three stocks' log-returns (Apple, Facebook, Google) more than the other direction. Second, the log-returns of Google stock were more affected by other three stocks' log-returns (Amazon, Apple, Facebook) than the other direction. By using the Kim and Hwang [8] copula directional dependence by SV model, we can understand how much US major tech stocks are related to each other, which can give some guidelines for consumers deciding which stock they need to buy first under a high volatility situation. If we generalize the [8] copula directional dependence by SV model to financial markets, then we can have a clear map about how the financial markets are related to each other. Readers can download the R code for the copula data analysis from the following website: http://cda.morris.umn.edu/~jongmink/research/QBIO.txt and can reproduce the parameter estimates for the copula data analysis easily.

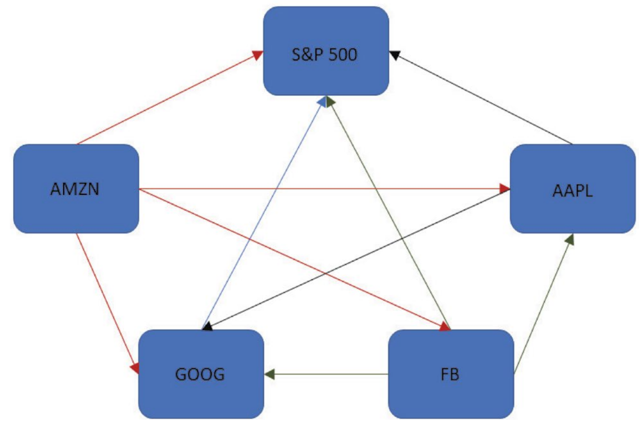

Fig. 3. Copula directional dependence of the log-returns of stock prices (period: January 3rd, 2020 to October, 22nd, 2020).

4. Copula Methods in Economics and Finance

We have reorganized copula methods in economics and finance based on two previous copula review papers ([48] and [49]). Since Patton [15] proposed a copula GARCH model based on the Engle [12] ARCH model and Bollerslev [13] GARCH model, numerous applications of copula theory in financial econometrics have been published in books and journals from economics, finance and statistics. Patton [48] reviewed the applications of copula methods for economic time series and classified the copula applications into five categories, which consisted of risk management, derivative contracts, portfolio decision problems, time-varying copula models, and high-dimension copula applications. Oh and Patton [50] modeled dependence in high dimensions with factor copulas of economic variables based on a latent factor structure to daily returns on all 100 constituents of the S&P 100 index and showed that factor copula models provide superior estimates of some measures of systemic risk. Other applications include using copulas to model dynamics in a panel of earnings data by [51] and using copulas to model the (uncorrelated) residuals of a multivariate GARCH model by [52]. Kim and Jung [53] applied copulas to dependence structure between oil prices, exchange rates, and interest rates. Liu et al. [54] investigated bank non-performing loans and government debt distress by copula tail dependence. Kim et al. [55] developed copula nonlinear Granger causality. But copula researchers knew that there was a limitation to extend parametric bivariate extreme value copula families to higher dimensions. Numerous copula researchers including Joe [56], Bedford and Cooke [57] and Kurowicka and Cooke [58] proposed different versions of copula methods to solve this limitation. Based on the previous researches of multivariate copula methods, Aas et al. [59] reorganized and proposed pair-copula constructions (PCC), which can construct the multivariate joint distribution by copula function. Since then, the application of PCC has been popular in financial economics. Berg and Aas [60] proposed models for the construction of multivariate dependence. Czado [61] proposed PCC of multivariate copulas. Min and Czado [62] proposed Bayesian inference for multivariate copulas using PCC. Smith et al. [63] modeled longitudinal data using a pair-copula decomposition of serial dependence. Czado et al. [64] proposed a maximum likelihood estimation of mixed C-Vines with application to exchange rates. Brechmann et al. [65] proposed truncated regular vines (an extension of PCC with vine dependence structure) in high dimensions with an application to financial data. Brechmann and Schepsmeier [66] developed the R-package CDVine, which is dependence modeling with C- and D-vine copulas. Pourkhandali et al. [67] also proposed to measure systemic risk using vine Copula.

Aas et al. [59] is one of the most highly cited papers in copula research area and it was cited 1,606 times on Google Scholar by October 31st, 2020. Aas [49] reviewed some of the PCC financial applications of market risk, capital asset pricing, credit risk, operational risk, liquidity risk, systemic risk, portfolio optimization, and option pricing. With a copula quantile method, Kim et al. [68] and Kim et al. [69] investigated the dependence structure of financial assets and the changing dynamics of board independence in corporate finance. For a better understanding of the vine copula, we recommend the vine copula website created by Professor Claudia Czado's research group Mathematical Statistics in the Department of Mathematics at the Technical University of Munich; https://www.groups.ma.tum.de/en/statistics/research/vine-copula-models/. The website consists of vine copula models research publications and open source implementations in R, C++ and Python, which are for vine copulas with time varying parameters, regime switching vine models, non-parametric vine pair copulas, Non-Gaussian directed acyclic graphical (DAG) models based on PCC, discrete vine copulas, truncated and simplified R-vines spatial vines, and copula discriminant analysis. The website is a useful site for researchers who are interested in the application of copula methods.

5. Conclusion

In this paper, we reviewed the copula-DCC-GARCH model, copula directional dependence and copula applications in economics and finance. To illustrate the copula-DCC-GARCH and copula directional dependence by SV model, we used US big tech FAANG stocks to show the time varying correlations between S&P 500 and FAANG stocks, and we drew the directional dependence map among US major stocks' log-returns. We also explained how much the copula time series models have been useful in economics and finance in the last two decades. Nowadays, uncertainty is a major research topic in economics, finance and many other areas. We can say that the copula may be a good statistical method for measuring the uncertainty with financial ultra high-frequency data.

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