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Forecasting Exchange Rate by Finding Trend Change

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ABSTRACT

In this article we propose a new type of time trend forecasting algorithm (TFA) which effectively incorporates a recent trend change into the forecaster by finding a proper trend from "recent length l segment" of exchange rate data. It will be shown that choice of l is critical for successful implementation of TFA and that predictive error-based method works pretty well for a proper choice of l. In addition monitoring tool is provided for checking the validity of the forecaster. An empirical study for exchange rate forecasting of Korea Won (KRW) to US Dollar (USD) is done for evaluation of the algorithm, which verifies usefulness of TFA particularly for yearly forecasting.

Key words : Exchange rate, Monitoring tool, Predictive-error based method, Recent trend change, Yearly forecasting

Introduction

Exchange rates occasionally exhibit dramatic changes in their behavior, associated with events such as financial crises or abrupt changes in government policy [1,2]. Thus it is widely accepted that empirical exchange rate models are characterized by dismal out-of-sample explanatory power or their poor forecasting performance [3,4]. Recently there has been increased interest in the models which could capture trend change or regime shifts effectively. As one of the outstanding models in this direction, [5] developed the well-known Markov switching model which assumes a finite number of models as the possible underlying trend and employs a discrete state Markov process for trend switching mechanism. [6] noticed that for many cases of exchange rates the Markov switching model (MSM) produces an excellent in-sample fit but still suffers from poor out-of-sample forecasts. Since then much efforts have been put into improving out-of sample forecasting performance of the Markov switching model (see [7-12]).

In this paper, we propose a new approach which utilizes trend change (or regime change) for forecasting in a different manner from MSM. Indeed assuming that time trend model is acting as the proper underlying forecaster (this will be justified later), the proposed trend forecasting algorithm (TFA) effectively incorporates a recent trend change into the forecaster. This is done by finding a proper trend from the "recent length l segment" of exchange rate data. It will be shown that choice of l is critical for successful implementation of TFA and that predictive error-based method might work pretty well for a proper choice of l. In addition monitoring tool is provided for checking the validity of the forecaster currently employed. An empirical study for exchange rate forecasting of Korea Won (KRW) to US Dollar (USD) is done for evaluation of the algorithm, which verifies usefulness of TFA particularly for yearly forecasting. This paper proceeds as follows. Section 2 proposes and discusses the time trend forecasting algorithm (TFA). Section 3 applies TFA to exchange rate forecasting for KRW to USD during 2004-2008 and evaluates its performance. Section 4 contains the concluding remarks.

Time Trend Forecasting Algorithm for Exchange Rate

1. Motivation

It is a well-established economic theory that the real exchange rate moves around the equilibrium exchange rate (EER) which

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is determined by demand and supply of foreign exchange market. Assuming the existence of EER, there could be two kinds of such, i.e., time invariant EER and time variant version [13]. The well-known time invariant EER is purchase power parity (PPP) model and it indicates the situation where EER is achieved invariantly over time thanks to constant macro economic equilibrium. In the meantime time variant EER indicates the case where EER moves dynamically as the macro economic equilibrium changes over time (see [14-16]).

When one examines the EER and the real exchange rates, he or she typically notices that the real rates move around EER slowly with a rather long cycle, often producing the graph of the two rates inter-wound together loosely (see [15]). This observation provides quite informative tips for employing time trend model for the exchange rate variation. Indeed the trend plus trigonometric function with a cycle L-day appears as a reasonable model for the exchange rate variation since the trend and the trigonometric function may be considered as representing EER and the (expected) cyclic difference between EER and the exchange rates, respectively. Thus, throughout this article, the time trend model

$$Z_t = f_0(t) + \alpha_1 \sin\frac{2\pi t}{L} + \alpha_2 \cos\frac{2\pi t}{L} + \varepsilon_t \quad \text{for } 1 \le t \le T \qquad (1)$$

is employed where Z_t is the exchange rate at time t, f_0 is assumed to be the EER, $\alpha_1 \sin\frac{2\pi t}{L} + \alpha_2 \cos\frac{2\pi t}{L}$ is the cyclic difference with the period L, ε_t's are assumed to be stationary errors and T is the present time. Of course an important issue for model (1) is estimating f_0, α_1, α_2 and L with exchange rate data. For this we will note the followings. First, we will consider the trigonometric function of (1) to be almost fixed once it is estimated properly from a larger set of the past data. As discussed earlier, this is based on the observation that cyclic movement of the discrepancy between the EER and the exchange rate tends to be time invariant. In other words cyclic mechanism between the government effort to correct the discrepancy and its effect to the rate is assumed to remain unchanged over time. Second, for estimating f_0 we will use the recent length l segment of exchange rate data and l is chosen by predictive error-based method. This step is based on the assumption that the trend recently changed after $t=T-l+1$ is valid for forecasting after $t=T$. Third, since we are mainly concerned about mid- or long-term exchange rate forecasting (see empirical studies in Section 3), exact inference about the stationary error with mean zero is not pursued in detail. Recall that the stationary error might play a significant role for daily forecasting and in such case it is to be fitted by a standard stationary process such as ARMA (autoregressive moving average). Finally note that for the case of time invariant EER, we may use model (1) by setting $f_0(t)=\mu$.

2. Time trend forecasting algorithm

For estimating model (1) at given current time T, time trend forecast algorithm (TFA) employs a simple linear trend $f_0(t)=\beta_0+\beta_1 t$ and fits model (1) to the recent exchange rate segment of length l. Now it is clear that the value of l totally determines the resulting forecaster and hence choice of l is critical for TFA. Our method develops the forecaster which focuses the recent segment of data for training and then use it as testing data for choice of l. This is done by introducing the forecaster which passes through the present data point but does not overfit to the recent length l segment of data.

For detailed description of TFA in the below, let us assume that $Z_1, \cdots, Z_T$ are given and we like to build the forecaster based on model (1) at the present time T. TFA consists of the three steps given as follows.

(i) Obtain a proper period L. For this, one may use the conventional spectral analysis applied to the past (de-trended) exchange rates of length l_s (>300) days or $Z_{T-l_s+1}, \cdots, Z_T$. In short, $\hat{L}$ might be searched over $0<L<l_s$ by spectral analysis. At this step expertise of economists might be of great help since L is basically related to long-term cyclic movement of discrepancy between the EER and the real exchange rates.

(ii) Estimate $\hat{\alpha}_1$ and $\hat{\alpha}_2$ with $(Z_1, \cdots, Z_c) \equiv (Z_{T-c+1}, \cdots, Z_T)$ for some $c \le l_s$. Note that from here and subsequently we use location transformation data $(Z_1, \cdots, Z_c)$ instead of $(Z_{T-c+1}, \cdots, Z_T)$ and hence the current data point Z_T corresponds to Z_c from now on. At this step we estimate $\hat{\alpha}_1$ and $\hat{\alpha}_2$ of (1) with $f_0(t)=\beta_0+\beta_1 t$, using $(Z_1, \cdots, Z_c)$ and $L=\hat{L}$. A standard least square method is employed here.

(iii) Use the recent length l segment of data to obtain an appropriate forecaster

$$\hat{f}_l(t) = \hat{\beta}_{0l} + \hat{\beta}_{1l} t + \hat{\alpha}_1 \sin\frac{2\pi t}{\hat{L}} + \hat{\alpha}_2 \cos\frac{2\pi t}{\hat{L}} \quad \text{for } c \le t \qquad (2)$$

where $\hat{\beta}_{0l}$ and $\hat{\beta}_{1l}$ are estimated with the recent l data $\mathbf{Z}_{c,l}=(Z_{c-l+1}, \cdots, Z_c)$. In particular the slope $\hat{\beta}_{1l}$ is estimated from $\mathbf{Z}_{c,l}$ first and then the intercept $\hat{\beta}_{0l}$ is obtained by equating

$$Z_c = \hat{f}_l(c). \qquad (3)$$

In other words $\hat{f}_l$ is forced to go through (c, Z_c). Recall Z_c denotes the exchange rate at the present. Now choose l^* such that for a positive integer $c_0 > 2$

$$l^* = \arg\min_{l \geq c_0} D(l) \tag{4}$$

where

$$D(l) = \sum_{t=1}^{l} (Z_{c-t} - \hat{f}_l(c-t))^2 / l.$$

Finally we have the adjusted forecaster

$$\hat{f}_{l^*}(t) = \hat{\beta}_{0l^*} + \hat{\beta}_{1l^*} t + \hat{\alpha}_1 \sin \frac{2\pi t}{\hat{L}} + \hat{\alpha}_2 \cos \frac{2\pi t}{\hat{L}_t} \quad \text{for} \quad t = c+1, \cdots. \tag{5}$$

For monitoring $\hat{f}_{l^*}(t)$ suppose that the residual $\{e_{t,l^*} = Z_t - \hat{f}_{l^*}(t): t = c - l^* + 1, \cdots, c, c+1, \cdots\}$ is a iid sequence and its marginal distribution is normal $N(0, \sigma^2_{l^*})$. Then it is easy to see that

$$Q_{l^*}(t) = \sum_{t-l^*+1}^{t} \frac{e^2_{t,l^*}}{\hat{\sigma}^2_{l^*}} \quad \text{asymptotically} \quad N(l^*, 2l^*) \tag{6}$$

where $\hat{\sigma}^2_{l^*}$ is an estimator of $\sigma^2_{l^*}$ from $\mathbf{e}_{c,l^*} = \{e_{c-l^*+1}, \cdots, e_c\}$. Thus $Q_{l^*}(t)$ could serve as a reasonable test statistics for H_0: no deviation from the current forecaster $\hat{f}_{l^*}$. Note that this procedure only detects a change caused by cumulative evidence and is unable to indicate exactly when the change started to occur.

3. Algorithm discussion

Remark 1. A key feature of TFA is that it effectively incorporates a recent trend change into the forecaster by finding a trend from recent length l segment of exchange rate data. Since TFA is designed for forecasting via a trend of length l recently changed, the accuracy of TFA totally depends on choice of l. For an appropriate selection of l (say l^*), we use test data approach. In fact $\mathbf{Z}_{c,l}$ used for training or obtaining $\hat{f}_l$ is also employed for testing for finding l^*. This is based on the idea that since the forecaster $\hat{f}_l$ itself is designed to avoid overfitting to $\mathbf{Z}_{c,l}$ for $l > c_0$ (see Remark 2 below), $\mathbf{Z}_{c,l}$ (and hence the predictive risk based on it) is expected to perform reasonably well as test data for choice of l. Note that the procedure for finding optimal l in (4) yields l^* achieving the minimum predictive risk which measures the average discrepancy between the forecaster $\hat{f}_l$ and $\mathbf{Z}_{c,l}$.

Remark 2. $\hat{f}_l$ employs a linear trend as EER (or $f_0(t) = \beta_0 + \beta_1 t$) and forces itself to pass through the present exchange rate (see (3)). In the meantime it imposes various conditions on itself. For example, (3), $l \geq c_0$ of (4), and other fixed estimates of $\hat{L}$, $\hat{\alpha}_{1l}$, $\hat{\alpha}_{2l}$ are imposed. Then from these one may notice the two things about the forecaster. First, passing through the present exchange rate forces the forecaster to focus on the most recent exchange rates regardless of value of l. In other words it produces an estimated sample path $\hat{f}_l$ that leads to (c, Z_c) for any l, which explicitly means that $\hat{f}_l$ weighs the most recent data including (c, Z_c) more importantly against the previous ones for any l. Second, $\hat{f}_l$ is designed to avoid overfitting to $\mathbf{Z}_{c,l} = (Z_{c-l+1}, \cdots, Z_c)$ for $l \geq c_0 > 2$. This is true because it is usually hard to overfit $\hat{f}_l$ to $\mathbf{Z}_{c,l}$ when a set of various conditions are imposed on $\hat{f}_l$. Indeed the conditions mentioned above contribute to preventing $\hat{f}_l$ from overfitting to $\mathbf{Z}_{c,l}$ (refer to Figs. 6 and 7 for a typical overfitting result without condition on $l \geq c_0$).

Remark 3. Linear trend is assumed for the forecaster, which eventually predicts either constant rise or fall of the rate when one uses the forecaster for a very long period of time. Thus it is very necessary to adjust the forecaster regularly or periodically. One may use the monitoring tool given at (6) for the adjusting purpose.

Remark 4. Though both TFA and Markov switching model (MSM) are developed for forecasting based on trend change, they differ mainly in two aspects. First, TFA focuses on finding "the current trend among infinitely many different trends" while MSM focuses on finding "the stochastic mechanism for transition among the finitely many different trends (two or three in most cases)". Indeed TFA examines various possible sample paths leading to the current exchange and then chooses one, which is done by selecting l^* from $l \geq c_0$ via (4), whereas MSM estimates the probability that the current trend is one of the given two or three trends, which is usually done by employing likelihood approach. Thus TFA has a much richer class of possible trends than MSM in finding a proper trend that governs the current exchange rate. This certainly gives a competitive edge to TFA for accurate forecasting. Second, TFA assumes the validity of the recent trend for the future while MSM assumes the validity of the stochastic trend change mechanism for the future. Thus forecasting performance of each algorithm critically depends on such assumptions. Here it is worth mentioning that checking the assumption for TFA is easy (see (6)) but checking the assumption for MSM is usually quite difficult (see [17]).

Empirical Examples

Throughout this section we consider the yearly forecasting of the exchange rate for KRW to USD. Indeed we use TFA to forecast the exchange rate for the coming year at the end of the current year, which will be done for the exchange rate data during 2004-2008 (see Fig. 1). Thus the c of algorithm description in Section 2-2 (see (2) or (3)) equals to 250 or 249 depending on the year (or c=250 or 249). This type of yearly forecasting is quite important since the government or the private firms used to make the next year budget at the end of the current year based on the forecasted exchange rate. Throughout this empirical study, $\hat{L}$=250 for step (i) is fixed because cyclic movement of the discrepancy between the EER and the real rate for KRW/USD is known to have period 250 roughly [15, 16]. In fact standard spectral analysis for the de-trended exchange rate during 2004-2008 confirms this (see Table 1). For simplicity of explanation, let $\hat{f}_{l,y}$ denote the forecaster estimated or trained on the recent l segment of exchange rate at $t=c$ (or at the end of year y) and define

$$D_y(l)=\sum_{t=1}^{l}(Z_{c-t}-\hat{f}_{l,y}(c-t))^2/l \tag{10}$$

$$Q_{l^*,y}(t)=\sum_{t-l^*+1}^{t}\frac{e^2_{t,l^*,y}}{\hat{\sigma}^2_{l^*,y}} \tag{11}$$

where $\{e_{t,l^*,y}=Z_t-\hat{f}_{l^*,y}(t)\colon t=c-l^*+1, \cdots, c, c+1, \cdots\}$ and $\hat{\sigma}^2_{l^*,y}$ is an estimator of $\sigma^2_{l^*,y}$ from $\mathbf{e}_{c,l^*,y}=\{e_{c-l^*+1,y}, \cdots, e_{c,y}\}$. Note the above quantities are introduced for defining (4)-(6) for our empirical study itself. Now for each year, we will do the followings. For obtaining α_1 and α_2 of step (ii), the standard least square method for (1) is employed with $Z_1, \cdots, Z_c$ and $\hat{L}$=250. To implement step (iii), l^*_y (the minimize of $D_y(l)$) is searched over l=50, 60 $\cdots$, 250 with c_0=50. For evaluation of the resulting yearly forecaster for the following year $y+1$ we calculate for l=50, 60 $\cdots$, 250,

$$D_{1,y+1}(l)=\sum_{t=1}^{c_1}(Z_{c+t}-\hat{f}_{l,y}(c+t))^2/c_1 \tag{12}$$

where c_1 is the number of the following year (or year $y+1$) exchange rates used for evaluation, usually c_1=250 since our yearly forecasting usually requires the length of year to be 250. Note that (12) is calculated to check whether l^*_y minimizing $D_y(l)$ is consistent with l^*_{1y} minimizing $D_{1,y+1}(l)$ (or whether the optimal choice from the previous year y performs reasonably well for prediction of the following year $y+1$). In addition, monitoring result by (11) is given for reference for each year below.

For forecasting 2005 exchange rate, the exchange rates from 2004.1.1-2004.12.31 are used. Figs. 2 and 3 depict $D_{2004}(l)$ (see (10)) and $D_{1,2005}(l)$ (see (12)) respectively, and the forecaster $\hat{f}_{l^*,2004}$ obtained from the 2004 exchange rates for 2005 forecasting is given in Fig. 4. In addition monitoring result is given in Fig. 5. From these figures one may notice the followings. Figs. 2 and 3 show that $D_{2004}(l)$ chooses l^*_{2004}=180 and $D_{1,2005}(l)$ (prediction evaluation) favors $l^*_{1,2005}$=250. However, note that advantage of $l^*_{1,2005}$=250 over l^*_{2004}=180, when being

Fig. 1. KRW/USD exchange rates from January, 2004 to June, 2008.

Table 1. Spectral analysis result for 2004-2007 exchange rates

L days (period)	Spectral estimate
.	3335.89
1107.00	3863.34
553.50	6196.98
369.00	8483.99
276.75	9767.38
221.40	8479.27
184.50	5534.94
158.14	2881.37
138.38	1325.91
123.00	1120.80
110.70	1118.59
100.64	1298.81
92.25	1088.59
85.15	784.93
79.07	481.62
73.80	356.22
69.19	264.42
65.12	276.85
61.50	448.64
58.26	680.51
55.35	787.04
52.71	695.66
50.32	560.22
48.13	419.41
46.13	298.95

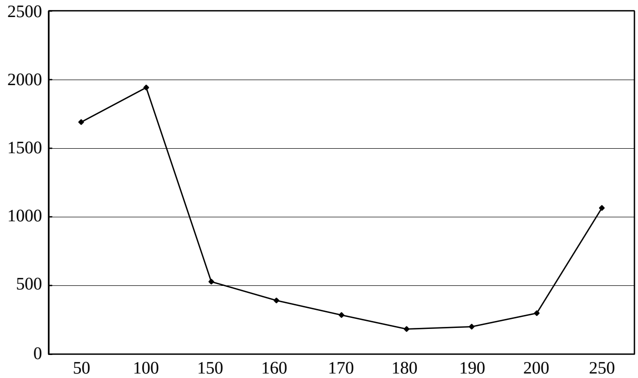

Fig. 2. Plot of $D_{2004}(l)$ which chooses $l^*_{2004}=180$.

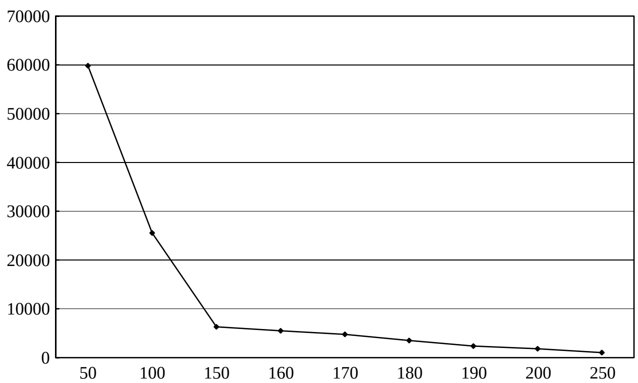

Fig. 3. Plot of $D_{1,2005}(l)$ which evaluates $\hat{f}_{l,2004}$ with the 2005 exchange rates.

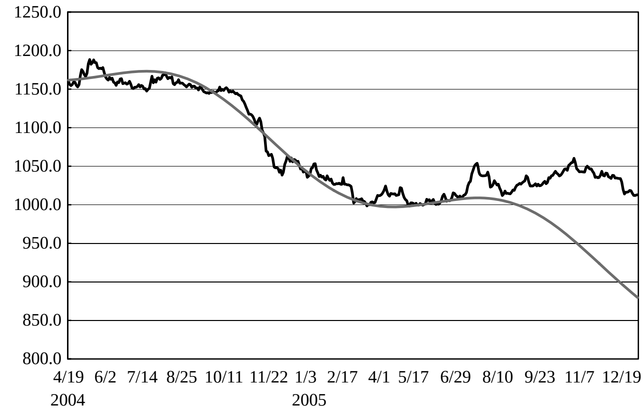

Fig. 4. Plot of $\hat{f}_{l^*,2004}=\hat{f}_{180,2004}$ (smooth curve) with the 2005 exchange rates.

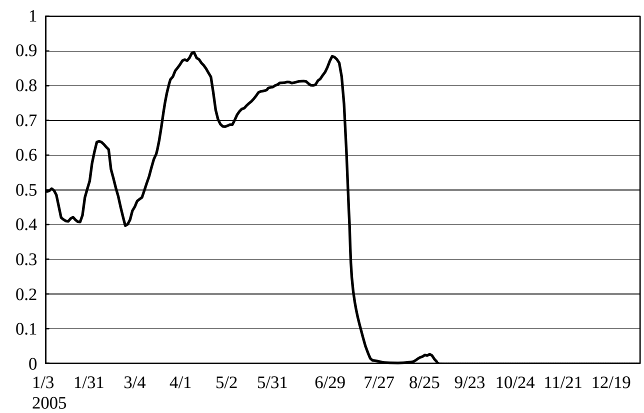

Fig. 5. Plot of p-value by test statistics $Q_{180,2400}(t)$ for 2005.

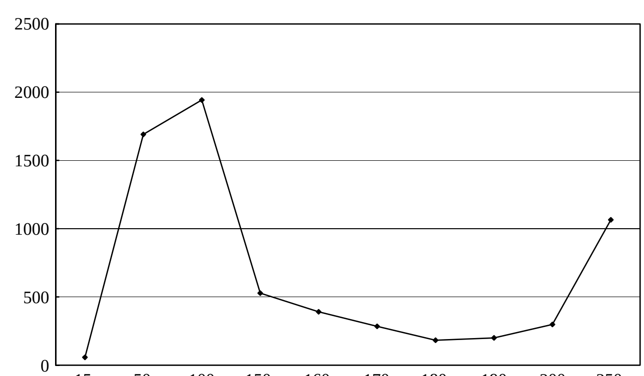

Fig. 6. Plot of $D_{2004}(l)$ with $c_0=15$.

evaluated via $D_{1,2005}(l)$, is not that significant as seen in Fig. 3. Fig. 4 depicts

$$\hat{f}_{l^*,2004}(t)=\hat{f}_{180,2004}(t)$$
$$=1224.636-0.639t-28.877\sin\frac{2\pi t}{250}-23.612\cos\frac{2\pi t}{250}$$

which is estimated from the 2004 exchange rates and shows that it performs reasonably well until about July of 2005 (or $t=410$). The monitoring result in Fig. 5 (or $Q_{180,2004}(t)$) picks up that time point successfully. Recall that a too small p-value indicates a failure of the forecaster $\hat{f}_{180,2004}$. In Figs. 6 and 7, we show that a proper choice of c_0 is necessary for successful implementation of TFA. Indeed Figs. 6 and 7 together show that $c_0=15$ may lead to a totally wrong choice of $l^*_{2004}=15$ and to overfitting to $\mathbf{Z}_{250,15}=(Z_{246}, \cdots, Z_{250})$.

For forecasting 2006 exchange rate, the exchange rates from 2005.1.1-2005.12.31 are used. Figs. 8 and 9 depict $D_{2005}(l)$ and $D_{1,2006}(l)$ respectively and the forecaster $\hat{f}_{l^*,2005}$ for 2006 is given in Fig. 10. Monitoring result is given in Fig. 11. From these results one may notice the followings. Figs. 8 and 9 show that both $D_{2005}(l)$ and $D_{1,2006}(l)$ equivalently choose $l=70$ or $l^*_{2005}=l^*_{2006}$. Also note that advantage of $l^*_{2005}=l^*_{1,2006}=70$ over other values of l is outstanding, particularly when being eva-

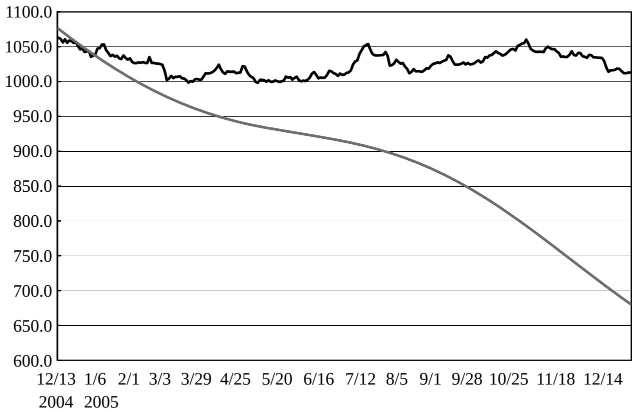

Fig. 7. Plot of $\hat{f}_{l,2004}=\hat{f}_{15,2004}$ (smooth curve) with the 2005 exchange rates.

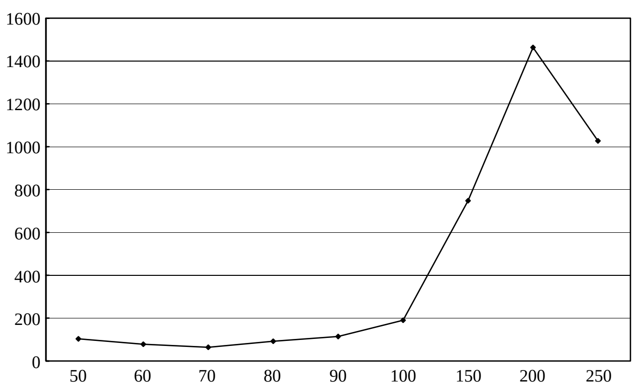

Fig. 8. Plot of $D_{2005}(l)$ which chooses $l^*_{2005}=70$.

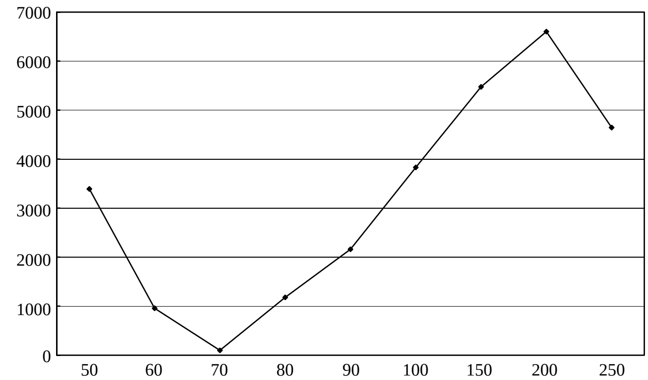

Fig. 9. Plot of $D_{1,2006}(l)$ which evaluates $\hat{f}_{l,2005}$ with the 2006 exchange rates.

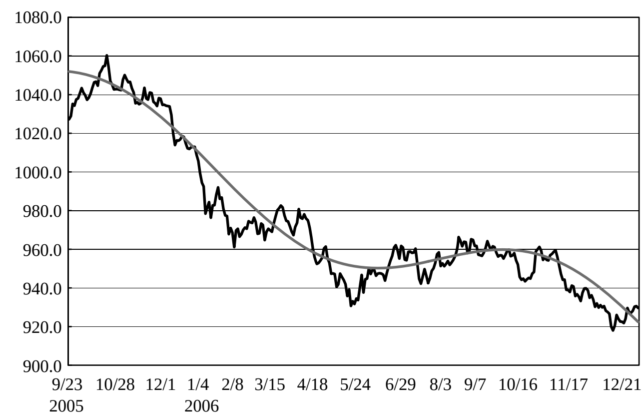

Fig. 10. Plot of $\hat{f}_{l^*,2005}=\hat{f}_{70,2005}$ (smooth curve) with the 2006 exchange rates.

luated via $D_{1,2006}(l)$ as seen in Fig. 9. Fig. 10 depicts

$$\hat{f}_{l^*,2005}(t)=\hat{f}_{70,2005}(t)$$
$$=1098.401-0.37t-21.949\sin\frac{2\pi t}{250}+7.029\cos\frac{2\pi t}{250}$$

for 2006 forecasting and shows that it performs reasonably well throughout the entire year 2006. The monitoring result ($Q_{70,2005}(t)$ Fig. 11) confirms this by indicating that model is valid except the period from $t=266$ (2006.1.27) to $t=377$ (2006.6.19).

For forecasting 2007 exchange rate, the exchange rates from 2006.1.1-2006.12.31 are used. Figs. 12 and 13 depict $D_{2006}(l)$ and $D_{1,2007}(l)$ respectively and the forecaster $\hat{f}_{l^*,2006}$ for 2007 forecasting is given in Fig. 14. The monitoring result is given in Fig. 15. From these one may notice the followings. Figs. 12 and 13 show that $D_{2006}(l)$ chooses $l^*_{2006}=150$ and $D_{1,2007}(l)$ (prediction evaluation) favors $l^*_{1,2007}=180$. However, note that advantage of $l^*_{1,2007}=180$ over $l^*_{2006}=150$, when being evaluated via $D_{1,2007}(l)$, is not that significant as seen in Fig. 13. Fig. 14 depicts

$$\hat{f}_{l^*,2006}(t)=\hat{f}_{150,2006}(t)$$
$$=973.1-0.188t-17.382\sin\frac{2\pi t}{250}+2.203\cos\frac{2\pi t}{250}$$

for 2007 forecasting and shows that it performs poorly, which is confirmed by the monitoring result ($Q_{150,2006}(t)$) in Fig. 15. The poor performance of the 2007 forecaster is expected to a certain degree since the estimated path of rates (or $\hat{f}_{l^*,2006}=\hat{f}_{150,2006}$) to the present rate Z_{250} (or the rate on the end day of 2006) seems to be an unlikely one in the sense that most of the exchange rates are below the estimated path near the end of 2006, as seen in Fig. 14.

For forecasting 2008 exchange rate, the exchange rates from

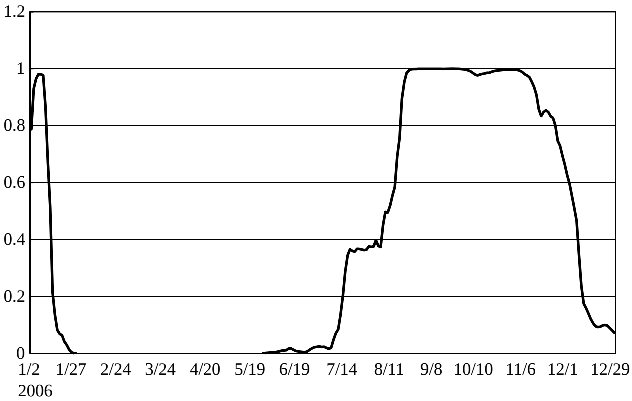

Fig. 11. Plot of p-value by test statistics $Q_{70,2005}(t)$ for 2006.

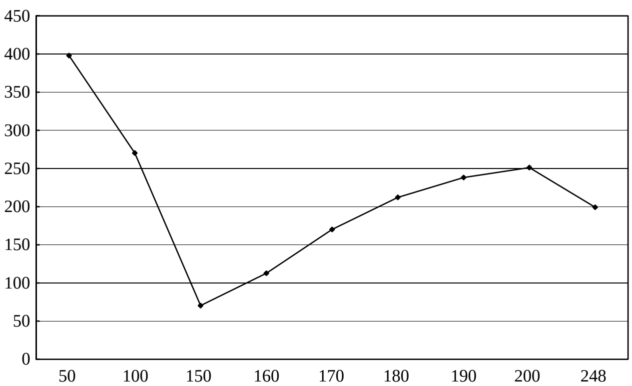

Fig. 12. Plot of $D_{2006}(l)$ which chooses $l^*_{2006}=150$.

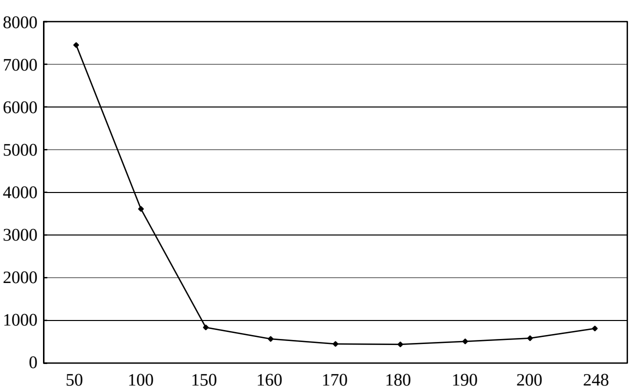

Fig. 13. Plot of $D_{1,2007}(l)$ which evaluates $\hat{f}_{l,2006}$ with the 2007 exchange rates.

Fig. 14. Plot of $\hat{f}_{l^*,2006}=\hat{f}_{150,2006}$ (smooth curve) with the 2007 exchange rates.

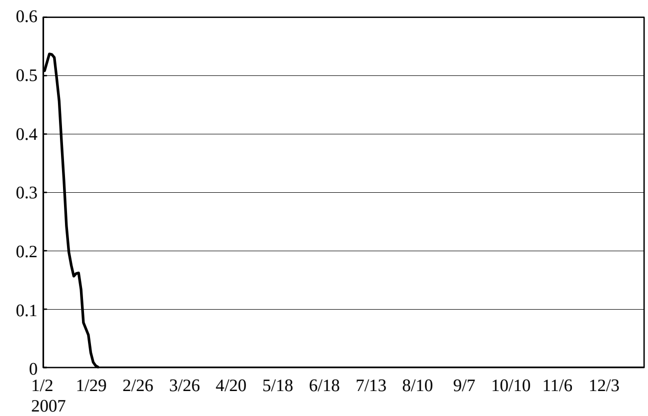

Fig. 15. Plot of p-value by test statistics $Q_{150,2006}(t)$ for 2007.

2007.1.1-2007.12.31 are used. Figs. 16 and 17 depict $D_{2007}(l)$ and $D_{1,2008}(l)$ respectively and the forecaster $\hat{f}_{l^*,2007}$ for 2008 forecasting is given in Fig. 18. The monitoring result is given in Fig. 19. From these figures one may notice the followings. Figs. 16 and 17 show that both $D_{2007}(l)$ and $D_{1,2008}(l)$ equivalently choose $l=50$ or $l^*_{2007}=l^*_{1,2008}$. Also note that advantage of $l^*_{2007}=l^*_{1,2008}=50$ over other values of l is outstanding, when being evaluated via $D_{1,2008}(l)$ as seen in Fig. 17. Fig. 18 depicts

$$\hat{f}_{l^*,2007}(t)=\hat{f}_{50,2007}(t)$$
$$=785.46+0.61t+3.721\sin\frac{2\pi t}{250}+2.374\cos\frac{2\pi t}{250}$$

for 2008 forecasting with the corresponding real exchange rates and shows that it performs reasonably well throughout the entire year 2006. The monitoring result $(Q_{50,2007}(t)$ of Fig. 19) indicates that $\hat{f}_{l^*,2007}$ fails to catch up the increasing trend after $t=284$ (or the end of February, 2008).

Remark 5. The year 2006 and 2008 KRW to USD exchange rate forecasting are the well-known cases where all forecasting by the private and government research institutes completely failed. Refer [16] for 2006 failure. For 2008 failure, it is well

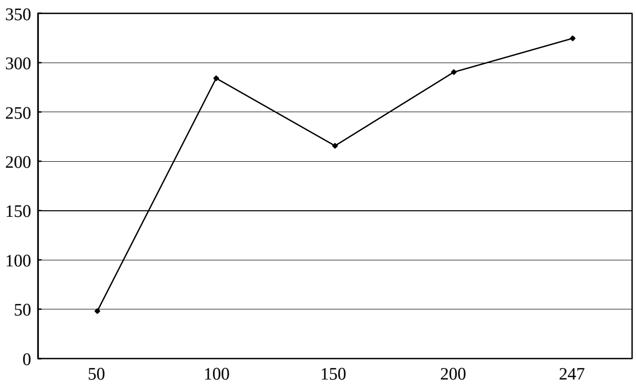

Fig. 16. Plot of $D_{2007}(l)$ which chooses $l^*_{2007}=50$.

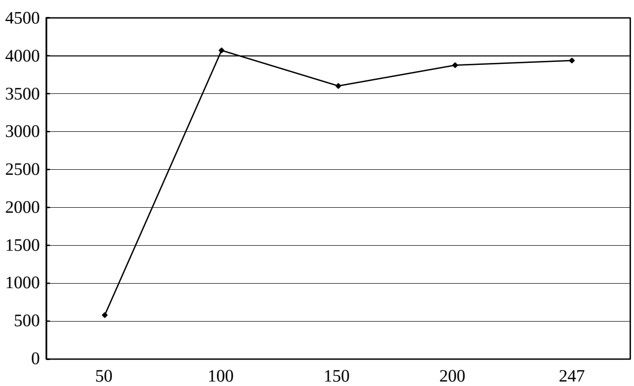

Fig. 17. Plot of $D_{1,2008}(l)$ which evaluates $\hat{f}_{l,2007}$ with the 2008 exchange rates.

Fig. 18. Plot of $\hat{f}_{l^*,2007}=\hat{f}_{50,2007}$ (smooth curve) with the 2008 exchange rates

known that no forecasting predicted the rising exchange rate in 2008 at all (see Fig. 18). Thus for each of two years their forecasting caused severe difficulty to the government or private firms carrying the following year budget based on their

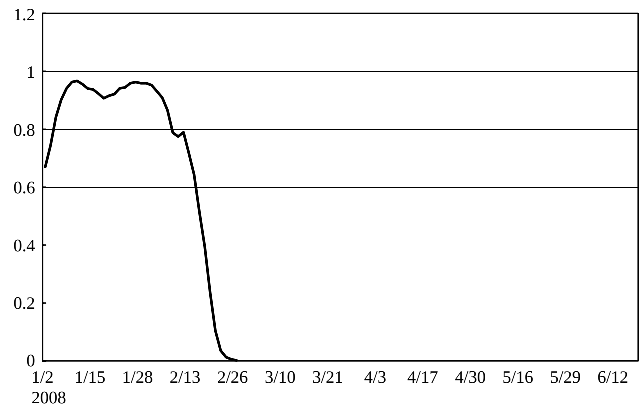

Fig. 19. Plot of p-value by test statistics $Q_{50,2007}(t)$ for 2008.

forecasting. Taking into account such complete forecasting failures, TFA here reports quite successful forecasting episodes for those years. On the other hand, it is worth mentioning that 2005 and 2007 forecasting by TFA registered failures, compared to the forecasting made for 2006 and 2008. These two different forecasting episodes clearly verify the advantages and the disadvantages of TFA. Indeed TFA is quite effective for finding the recent trend change made before $t \le c-c_0$ where c is the current time point (recall that the condition $l \ge c_0$ is imposed for preventing overfitting of the forecaster). However, if the trend change occurs $c-c_0<t$ and that trend governs the future, failure of TFA is inevitable. For example, one may easily see that the year 2006 (i.e., failure year) has time variant EER before $c-c_0$ but trend has changed to an almost time invariant EER after $c-c_0$ (Fig. 14). Similar remarks could be applied for 2005 forecasting.

Concluding Remarks

This paper proposes the TFA algorithm which incorporates recent trend change into the forecaster effectively. As seen in empirical examples, it works quite well as long as the recent trend change starts before reasonably earlier than the present time. It is interesting to report that TFA could improve MFM in two aspects; (i) it considers various types of trend changes, which leads to an improved forecasting performance (ii) it is easier to check validity of TFA than MFM. Also note that it is usually hard for general econometric forecasting model to incorporate the recent trend change into the model since it requires timely adjustments of the various related economic variables and their relations in the model.

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